

## Folding of passive layers and forms of minor structures near terminations of blind thrust faults—application to the central Appalachian blind thrust

PHILIP BERGER and ARVID M. JOHNSON

Department of Geology, University of Cincinnati, Cincinnati, OH 45221, U.S.A.

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**Abstract**—The existence of detachment surfaces or décollement zones beneath folded rocks of the Valley and Ridge and Plateau provinces of the Appalachians has been recognized as an important condition of folding. Large folds at the border between the two provinces resulted primarily from repetition of strata by thrusting of blocks over ramp faults that connect detachment surfaces at different horizons. Some investigators have suggested that folds in the Plateau province of Pennsylvania were produced by splay faults arising from detachment surfaces, but field observations and theoretical analyses by Sherwin and by Wiltschko & Chapple suggest that the folds are a result of buckling of multilayered rocks above a décollement. An exception may be the Burning Springs anticline in West Virginia, which appears to have formed at the termination of a detachment surface.

Investigation of the translation of an homogenous, viscous material above a flat detachment surface that terminates laterally indicates that the termination produces a broad, low-amplitude anticline in passive layering as a result of thickening induced by a gradient of shear stresses in the vertical direction. This thickening above the termination of a detachment is a mechanism of folding. If the viscous fluid contained mechanical layering, the fold would become amplified by buckling. Computations of stresses in the material indicate that minor faults should be generated first near the termination of the flat detachment surface. The Burning Springs anticline probably was initiated by termination of a detachment surface and subsequently amplified by buckling.

### INTRODUCTION

There has been considerable progress toward mechanical descriptions of conditions of folding and faulting in the Valley and Ridge and adjacent Plateau provinces of the central and southern Appalachian mountains. A major step in the development of mechanical analyses was the recognition of detachment surfaces and of zones of décollement at various levels in the stratigraphic section along which blocks of overlying rock have been pushed westward. This recognition provided an explanation for differential lateral shortening at different levels in the section. Progress has also been made towards understanding the various styles of deformation of blocks that have been translated and shortened, including imbricate faults and décollement folding.

Above detachment surfaces in lower Palaeozoic rocks in the southern Appalachians the rocks have been predominantly dislocated along imbricate thrust faults (Rodgers 1949, King 1972, Roeder *et al.* 1978). Similarly, detailed maps and cross-sections of the anthracite coal fields in east-central Pennsylvania show that imbricate thrusting was an important mechanism of accommodating horizontal shortening of Pennsylvanian rocks in that area (Wood *et al.* 1969). Some aspects of the conditions of imbricate faulting were investigated by Chapple (1978), who idealized the rock sequences as wedges of rigid plastic material bounded below by a detachment surface and above by a stress-free surface. Chapple determined slopes of the ground surface and strengths along the detachment surface for which the wedge of plastic could be in a state of yield. The slip surfaces in the yielding plastic material can be considered to be analogous to imbricate faults in rock. Chapple

provided an important mechanical idealization of imbricate faulting that can be a basis for mechanical analyses of specific complexes of imbricate thrusts.

Wiltschko & Chapple (1977 p. 668), in a discussion of folds on the plateau in Pennsylvania, suggested that a thick, soft décollement zone in western Pennsylvania allowed folding to occur there, but that such a zone was missing at the appropriate location in the stratigraphic section of the southern Appalachians to have allowed folding there. Johnson (1980) suggested a complementary explanation for the predominance of faulting in the southern Appalachians, based on analysis of folding of single- and multi-layers of a variety of sedimentary rocks. He approximately modelled the behaviours of rocks tested in the laboratory by means of elastic-plastic strain-hardening rheology and showed that rocks with these properties would tend to fold or fault when subjected to layer-parallel compression depending primarily upon the number of structural layers comprising a multilayered rock sequence; where there were few structural members, the rocks would fault before they could fold significantly, whereas where there were many structural members, the rocks would tend to fold. Thus a thick, soft zone of décollement and many structural members favour buckle folding.

The existence of detachment surfaces or zones of décollement beneath the gently-folded rocks of the Plateau Province in western Pennsylvania and West Virginia has been recognized as an important condition of folding there (Rodgers 1949, 1950, 1963, Gwinn 1964, 1970, Woodward 1959, Prucha 1968, Sherwin 1972, Wiltschko & Chapple 1977). The folds on the plateau in western Pennsylvania generally have parallel fold axes, wavelengths of the order of 8–16 km and low structural

relief. The anticlines are generally asymmetric, but the steeper limb occurs in the east in some areas and in the west in others. The cores of most of the folds contain symmetric thrust faults, so that faults generally dip eastward on the east limb and westward on the west limb (Rodgers 1963, Gwinn 1964). Rodgers (1963) suggested that the anticlines were passive structures, resulting from repetition of strata by thrusting within the cores of the folds. Gwinn (1964) suggested that the anticlines formed before the thrusting in the core but that the thrusting was largely responsible for the structural relief of the anticlines. Both interpreted the thrusts in the cores of folds as splay faults connecting with detachment surfaces at depth. Sherwin (1972), however, pointed out that structural interpretations of folds on the plateau based solely on gliding of blocks along detachment surfaces did not explain the regular wavelengths or the development of thrusts in the cores of the anticlines (e.g. Rodgers 1963). She analysed the folding of simple multilayers above zones of décollement and used the theoretical results to explain some features of the Firtree Point and the Chestnut Ridge anticlines. She showed that a variety of multilayered models of the behaviour of the rocks could account for the observed wavelengths of folds, the occurrence of zones of intense deformation and the lack of deformation in rocks at or above the basement rocks. The zones of intense deformation corresponded to a zone of flowage of salt in the Firtree Point anticline and the zone of symmetric thrusting in the Chestnut Ridge anticline, so that the symmetric thrusting, in her view, was largely a result of folding rather than the cause of folding.

Wiltshko & Chapple (1977) re-examined the folds on the plateau in Pennsylvania and concluded that only part of the structural relief of the folds was accounted for by symmetric thrusting, and that there must have been a flow of relatively soft materials into cores of anticlines from adjacent synclines. They cited several field examples of decreased thickness of soft units beneath synclines and increased thickness beneath anticlines. Thus, theoretical analyses and field data presented by Sherwin and by Wiltshko & Chapple indicate that the folds are a result of westward sliding of blocks over detachment surfaces or zones of décollement, but that the folds grew primarily by buckling rather than by repetition of strata above symmetric thrust faults that connect as splay faults to detachment surfaces.

However, some large folds at the border between the Valley and Ridge and the Plateau Provinces have been produced primarily by repetition of strata by thrusting of blocks over ramp faults connecting detachment surfaces at different levels. Powell Valley anticline in the southern Appalachians was early recognized to be of this type (Rich 1934), and has become essentially the type example of a ramp fold (Rogers 1949, 1963, Gwinn 1964, 1970, Harris 1970, Harris & Milici 1977, Wiltshko 1979a). Subsequently, several other folds of this type have been recognized, including the Nittany arch (Gwinn 1964) and the Wills Mountain anticline (Perry 1978), in the central Appalachians of Pennsylvania and West Virginia, and Sequatchie Valley anticline (Rodgers 1950) on the Plateau

Province of the southern Appalachians.

Analyses of folding or deformation in rocks that have been thrust over ramps between detachment surfaces have been made by Raleigh & Griggs (1963), Elliott (1976a, 1976b), Wiltshko (1979a, 1979b) and Berger & Johnson (1980). Raleigh & Griggs performed a modification of the analysis of Hubbert & Rubey (1959), which showed effects of pore-water pressure on reducing resistance to sliding on a detachment surface, by incorporating a ramp as a toe of the ideal thrust sheet, and approximately analysing the resistance of the toe. Elliott (1976a) incorporated effects of roughness of the detachment surface in the manner introduced by glaciologists (Collins 1968, Nye 1969, Budd 1970a, 1970b, Kamb 1970, Morland 1976a, 1976b, McClung 1981), and estimated driving and resisting forces acting on idealized thrust sheets (1976b). He did not specifically introduce ramps in the detachment surface. Wiltshko (1979a, 1979b) determined stresses and deformations in a viscous layer moving over a ramp and analysed the drag caused by the ramp. He treated the rock beneath the detachment as rigid and the overriding block as an anisotropic, viscous plate, subjected to uniform rate of shortening and to translation of the plate over the ramp. He concluded that resistance to bending of the overriding plate was as important as resistance to drag on the detachment surface, and that normal and reverse faults in the overriding plate would become superimposed because the plate was subjected to positive and then to negative bending as it rode over the ramp. He also concluded that movement of thrust sheets required both horizontal compression and topographic slope toward the distal end of the thrust sheet.

In a previous paper (Berger & Johnson 1980), we investigated the form of folds in passive layers in an homogeneous viscous thrust sheet translated over an idealized ramp in a detachment surface, and showed that the folds would be flat-topped and have symmetric limbs much as theorized by Rich (1934). Further, based on a study of the effects of adhesional drag between the base of the thrust sheet and the surface of the ramp, we concluded that an asymmetric fold similar to the Powell Valley anticline could be produced if a fold at the ramp occurred first, as a result of buckling, splay faulting or drag on the ramp, and if the fold was then translated over the ramp.

The folds in the Valley and Ridge Province in the central Appalachians of Pennsylvania have traditionally been interpreted to be repetitious folds caused by buckling by layer-parallel compression (Willis 1893), and the fold forms were interpreted to be concentric-like (e.g. Chamberlin 1910). Nickelsen (1963) described the folds in Silurian and Devonian rocks as sinusoidal forms with straight limbs and tight hinges (i.e. as chevron folds). Faill (1969, 1973) redescribed the folds throughout the central Appalachians as intersecting kink bands, but geometric analysis of outcrop patterns of plunging folds of two large synclines and one anticline strongly suggests that the forms are combinations of concentric-like and chevron folds; indeed, large kink forms are absent (Johnson 1981). Further, the sense of asymmetry of small folds on the

limbs of the large folds indicates that the small folds are drag folds, not kink bands (Reches & Johnson 1976). Combinations of concentric-like and chevron folds generally are interpreted to be a result of buckle folding (Johnson & Honea 1975, Johnson 1977). Gwinn (1970) suggested that the large folds in the Silurian and Devonian rocks are concentric forms but that folding was produced largely by splay faults connecting with a major detachment surface in Cambrian rocks.

Thus, folds in the southern Appalachians, along the Allegheny Front in the central Appalachians, and in Pennsylvanian rocks in the anthracite district of east-central Pennsylvania, appear to result predominantly from repetition of strata by imbricate splay faulting or by thrusting over ramps. Folding on the plateau of Pennsylvania appears to be largely a result of buckle folding of relatively competent rocks above soft zones of décollement. Folding in Silurian and Devonian rocks in the Valley and Ridge Province in central Pennsylvania appears to be a result of buckling, although the role of splay faulting in growth of the folds remains unclear.

In this paper, we investigate folding of passive layering in an homogeneous viscous material near the termination of a flat detachment surface (blind thrust, Thompson 1979). The motivation for the study is that such a mechanism of folding has not been investigated analytically, although it has been suggested to be responsible for some folds. For example, the Burning Springs anticline in western West Virginia and eastern Ohio may have formed partly as a result of termination of a detachment surface (Woodward 1959). The Burning Springs anticline is somewhat different from many folds on the Plateau; it trends north-south, rather than about N 45° E as is typical, and it has higher structural relief at the ground surface, about 500 m, than folds nearby (Woodward 1959, Rodgers 1963). According to Rodgers (1963), the Burning Springs anticline is a flat-topped fold with steep sides; however, cross-sections by Cardwell *et al.* (1968) show the anticline to be a rounded, asymmetric form, with the steeper limb facing eastward. According to Woodward (1959), a well drilled through the fold showed a normal stratigraphic section to a depth of about 1.2 km, near the top of the Oriskany Sandstone; but below that depth, several units, comprising the interval between Onondaga and Helderberg formations, were repeated by imbricate thrusting, producing an increase of section of about 500 m in excess of normal thicknesses of the units. Below the faulted zone the Lower Devonian rocks apparently are flat-lying. The Late Silurian rocks of the upper Salina Group encountered in the well contain anhydrite, but no salt. According to Rodgers (1963 p. 1533), a well drilled about 6 km east of the Burning Springs anticline encountered about 30 m of salt in the upper Salina Group, and evidence from wells west and north of the anticline indicate that the salt pinches out in the vicinity of the anticline. Rodgers attributed the thrusting in the core of the anticline to lateral pinching out of the salt. Gwinn (1964 p. 878) suggested that, "... increase of frictional resistance along the décollement west of the salt wedge-out impeded westward movement

of the upper part of the plateau cover, producing imbricate thrust faulting; the ... anticline was produced as an incidental effect of the thrusting at depth." Rodgers (1963) suggested essentially the same mechanism of folding. Thus, the fold apparently was initiated by termination of a detachment surface or a zone of soft salt, and subsequent faulting was the mechanism by means of which rocks in the core of the anticline were thickened.

In this paper we show that terminations of detachment surfaces can produce passive folds by thickening the section locally. Our analysis should be relevant to early stages of formation of folds similar to the Burning Springs anticline. However, it is possible that, once initiated, most of the growth of the Burning Springs anticline was a result of buckle folding. As suggested by Sherwin (1972) for the Chestnut Ridge anticline, the faulting in the core of the Burning Springs anticline may have been a mechanism of thickening of the section there during buckle folding of overlying rocks, rather than the cause of the folding as suggested by Rodgers (1963) and by Gwinn (1964).

The analysis presented here is similar to that developed to describe folding of passive layers over idealized ramp faults (Berger & Johnson 1980). There, we considered the effect of adhesional drag, represented by constant shear stress, on a ramp connecting a lower detachment surface to a higher detachment surface. Drag was assumed to be zero along the flat detachment surfaces. Here, we investigate deformation in viscous material above a flat detachment surface along part of which adhesional drag is large and along part of which drag is zero (Fig. 1a).

We compare the folding caused by the termination of a flat detachment surface with the folding caused by repetition of strata by a ramp fault connecting frictionless detachment surfaces at two different levels as discussed by Berger & Johnson (1980). We examine deformation associated with a more complicated detachment surface

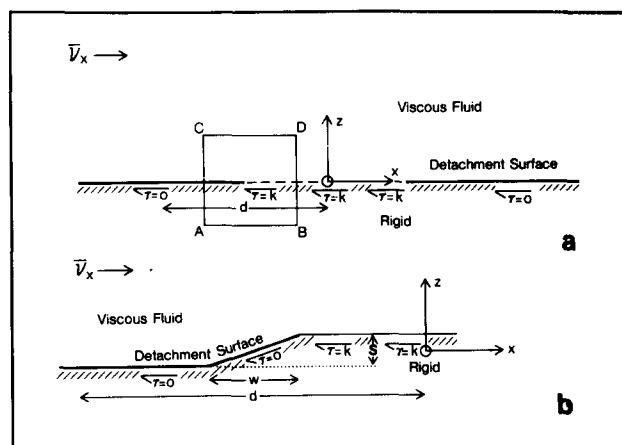


Fig. 1. Shape of detachment surface and distribution of drag assumed in theoretical analyses. (a) Symbols used to describe the flat detachment surface. Area ABCD denotes region near the termination of the detachment surface shown in Fig. 2. Solid heavy line is segment with free slip. Dashed line is segment with drag of magnitude  $k$ . (b) Symbols used to describe the stepped detachment surface which is comprised of a dipping ramp segment connecting the upper and lower flat segments. Heavy line is segment with free slip (lower flat segment and ramp segment). Drag on upper flat segment of magnitude  $k$ .

that terminates at the top of a ramp fault (Fig. 1b), where folding is caused both by drag and by repetition of strata. The ramp fault connects a lower detachment surface with zero drag to an upper detachment surface with high adhesive drag.

### SOLUTION

In order to determine velocities and stresses near the termination of a detachment surface, we analyse a model that assumes rigid material below the detachment and viscous material above (Fig. 1a). The detachment surface is assumed to be frictionless so that the horizontal shear stress is zero there. The viscous material over the extension of the detachment surface (dashed line in Fig. 1a) is assumed to adhere to the rigid base, and the maximum shear stress the base can exert onto the contacting viscous fluid is the adhesional strength,  $k$ .

We use Fourier series in order to solve this problem, so that the detachment surface and the adhesional surface, both of width  $d$ , are repeated cyclically along the  $x$ -axis. We present results of calculations for a small area, ABCD (Fig. 1a), of the body; so for translations that are a small fraction of  $d$ , the results are similar to those one would obtain for infinitely long detachment and adhesional surfaces. The method of solution of such problems has been well documented (Kamb 1970, Fletcher 1977, Reches & Johnson 1978, McClung 1981), so we will omit discussion of the method here.

The solution for the velocities is,

$$v_x = \bar{v}_x + (kz/2\eta) + \sum_{m=1}^{\infty} (R_m/2\eta l_m) (l_m z - 1) \times \exp(-l_m z) \cos(l_m x) \quad (1a)$$

$$v_z = - \sum_{m=1}^{\infty} (R_m/2\eta l_m) (l_m z) \exp(-l_m z) \sin(l_m x) \quad (1b)$$

in which

$$l_m = m\pi/d. \quad (1c)$$

Equations (1a) and (1b) are valid for any distribution of adhesional drag that can be expressed by a Fourier cosine series. For constant adhesional drag, we have assumed,

$$R_m = (2k/m\pi) \sin(m\pi/2); \text{ odd } m. \quad (1d)$$

The velocity in the  $x$ -direction ( $v_x$ ), equation (1a), is the sum of three contributions. The first term in equation (1a), ( $\bar{v}_x$ ), is a constant and defines uniform translation of the fluid. The other two contributions are the result of the drag along the detachment surface. The drag causes a mean shear flow [second term in equation (1a)] and a perturbed shear flow (third term). The mean shear flow,  $kz/(2\eta)$ , where  $\eta$  is the viscosity of the fluid, is produced by

the average stress,  $k/2$ , along the interface between the fluid and the rigid substratum. We assume that the mean shear flow is zero at the detachment. It increases linearly with  $z$ , so that far from the interface the velocity is a combination of the uniform translation and the mean shear flow. The perturbed shear flow, the third contribution to the velocity, decreases exponentially with increasing  $z$  and is required to satisfy the boundary conditions along the detachment surface. The velocity in the  $z$ -direction ( $v_z$ ) is solely a result of the perturbed shear flow. It decreases exponentially with increasing  $z$ .

In evaluating the solution we determine displacements of points by integrating numerically over time. We introduce a drag parameter,  $K$ , which is proportional to the ratio of the adhesional drag,  $k$ , and a characteristic shear stress in the fluid,  $(\bar{v}_x/d)\eta$ , such that

$$K = (kFd/\bar{v}_x\eta) \quad (2a)$$

where  $F$  is a factor, the value of which is determined as follows. For  $K = 1$ , we set the velocity equal to zero at  $x = 0$ ,  $z = 0$ , in equation (1a).

$$\bar{v}_x = \sum_{m=1}^{\infty} (R_m/2\eta l_m). \quad (2b)$$

Substituting (2b) and (1d) into (2a),

$$F = \sum_{m=1}^{\infty} (1/m\pi)^2 \sin(m\pi/2); \text{ } m \text{ odd}. \quad (2c)$$

Thus, for  $K = 1$  there is no slippage at point (0, 0), but there is slippage everywhere else along the interface. For  $K$  less than one there is slippage everywhere, and for  $K = 0$  there is zero adhesional drag everywhere. We considered a range of  $K$ -values elsewhere (Berger & Johnson 1980), but here we will assume  $K = 1$  for all calculations.

The solution is for a homogeneous viscous fluid, but we can qualitatively use the solution to determine types of minor folds and local faults that might develop in the stress field produced by termination of the detachment surface. We compute the stress difference and shear stress by means of the velocities, equations (1):

$$(\sigma_{xx} - \sigma_{zz}) = 4\eta(\partial v_x/\partial x) \quad (3a)$$

$$\sigma_{xz} = \eta[(\partial v_x/\partial z) + (\partial v_z/\partial x)]. \quad (3b)$$

The mean pressure is

$$\begin{aligned} \bar{p} &= -(\sigma_{xx} + \sigma_{zz})/2 \\ &= - \sum_{m=1}^{\infty} R_m \exp(-l_m z) \sin(l_m x) \end{aligned} \quad (3c)$$

assuming a zero value at  $x = 0$ .

In order to discuss possible orientations of minor faults in the material we assume that the condition of faulting is described by the generalized Coulomb criterion (Rudnicki & Rice 1975):

$$\sqrt{J_2} - \bar{p} \tan(\phi) \leq C + \bar{p}_0 \tan(\phi) \quad (4a)$$

according to which yielding (faulting) occurs if the

equality is satisfied. Here  $\tan\phi$  is the coefficient of friction, and  $\phi$  is the friction angle, which we assume to be  $30^\circ$  for the computations.  $J_2$  is the second invariant of the deviatoric stresses which, for plane strain, is,

$$J_2 = [(\sigma_{xx} - \sigma_{zz})/2]^2 + (\sigma_{xz})^2 \quad (4b)$$

and  $\bar{p}_0$  is the overburden mean pressure, which must be added to the mean pressure computed via equation (3c).

In order to discuss where minor kink bands and drag folds might form in the fluid, we assume that a layered fluid is subjected to the stresses computed for the homogeneous fluid. The relevant stresses for folding are the normal stresses parallel,  $\sigma_{ss}$ , and normal,  $\sigma_{nn}$ , to markers between layers, and shear stress,  $\sigma_{ns}$ , parallel to layers. If  $\theta$  is the slope angle of layering,

$$\sigma_{ss} - \sigma_{nn} = (\sigma_{xx} - \sigma_{zz}) \cos(2\theta) + 2\sigma_{xz} \sin(2\theta) \quad (4c)$$

$$\sigma_{ns} = [(\sigma_{zz} - \sigma_{xx})/2] \sin(2\theta) + \sigma_{xz} \cos(2\theta). \quad (4d)$$

This completes the set of equations required to compute velocities, displacements, strains and stresses in the viscous fluid in the vicinity of the termination of a detachment surface.

#### DEFORMATION NEAR THE TERMINATION OF A DETACHMENT SURFACE

Now let us examine results of computations of displacements, strains and stresses in a viscous fluid with passive layering near the termination of a flat detachment surface. There is zero drag along the detachment surface but along the extension of the detachment surface there is sufficiently great adhesional drag to prevent the fluid at the origin of coordinates (Fig. 1a) from moving. In the results presented in Fig. 2, the viscous fluid has been translated one-tenth of the length of the frictionless detachment surface (0.1 unit).

Figure 2(a) shows final positions of interfaces between passive layers in the viscous fluid that were originally horizontal. The interfaces have been deformed into a broad, low-amplitude fold as a result of translation and drag. The drag itself is reflected in the tilting of the steeply inclined lines at either end of the figure; these lines originally were vertical. The layer-parallel shear increases from zero at the frictionless detachment surface at the left-hand end of the figure to a larger value at the top of the figure, as reflected in the tilt of the end line. In contrast, the layer-parallel shear decreases from a maximum value at the surface with large adhesional drag at the right-hand side of the figure, to a smaller value at the top of the figure.

The amplitude of the fold in the passive layering increases from zero at the detachment surface to a maximum value at some distance above the detachment surface, and then decreases to zero at very large distances above the detachment surface. The distance above the detachment surface at which folding will be negligible will depend upon the amount of displacement on the detachment surface; the greater the displacement, the greater the distance of significant folding above the

detachment surface. The width of the fold increases upward, above the detachment surface, so that the width will increase and the amplitude will decrease with increasing distance above the detachment surface.

Finally, the culmination of the fold shifts in the direction of transport with increasing displacement and with increasing distance above the detachment surface.

The mechanism of the folding in the passive layers shown in Fig. 2(a) is the reduction in horizontal velocity near the termination of the detachment surface. The formation of the fold reflects the gradient in shear stresses in the vertical direction; the equation of equilibrium indicates that a change in shear stress in the vertical direction is counteracted by a change in normal stress in the horizontal direction. Thus, the drag causes the viscous fluid to be compressed parallel to layering in the vicinity of the termination so that the viscous fluid thickens there, producing the fold. The folding in this case is a result of a fundamentally different mechanism than that described by Rich (1934), where folding is caused by repetition of stratigraphic section over frictionless detachment surfaces and ramps (Fig. 3a).

Directions and magnitudes of maximum finite compressive strains are shown in Fig. 2(b). The directions range from being parallel to the detachment surface at the detachment surface itself to being at an angle of slightly greater than  $45^\circ$ , plunging distally, in the overburden overlying the surface of high drag. The magnitude of the finite compressive strains are indicated in per cent, that is, in terms of  $(1 - L_f/L_0) \times 100$ , where  $L_f$  is the final length and  $L_0$  is the original length of a line element. For far-field translation of one-tenth of the length of the detachment surface, the maximum finite shortening is about 30% near the termination. The compressive strains are larger in the fluid over the surface of high drag than they are over the detachment surface.

Magnitudes of the difference between layer-parallel and layer-normal stresses, divided by adhesional drag,  $k$ , and senses of layer-parallel shear are indicated in Fig. 2(c). Positive values of the stress difference indicate layer-parallel compression. The solution, of course, was derived for an homogeneous, isotropic fluid, so that buckle folding cannot occur. However, we can use the solution to extrapolate where buckle folds would form and to indicate the sense of asymmetry of minor buckle folds if the fluid were layered. The sense of asymmetry of drag folds and monoclinical folds are opposite for the same state of deformation (Reches & Johnson 1976, Ramberg & Johnson 1976). Asymmetric chevron folds and drag folds will form with their short limbs facing distally, and monoclinical kink bands will form with their short limbs facing proximally, as shown in the inset sketches in Fig. 2(c). The stress difference is maximal near the termination of the detachment surface, so the strongest tendency for the minor folds to form is there.

Figure 2(d) shows directions of minor faulting and magnitudes of a yield function, which is based on the generalized Coulomb law of failure and is the value of the cohesive strength plus the overburden mean pressure

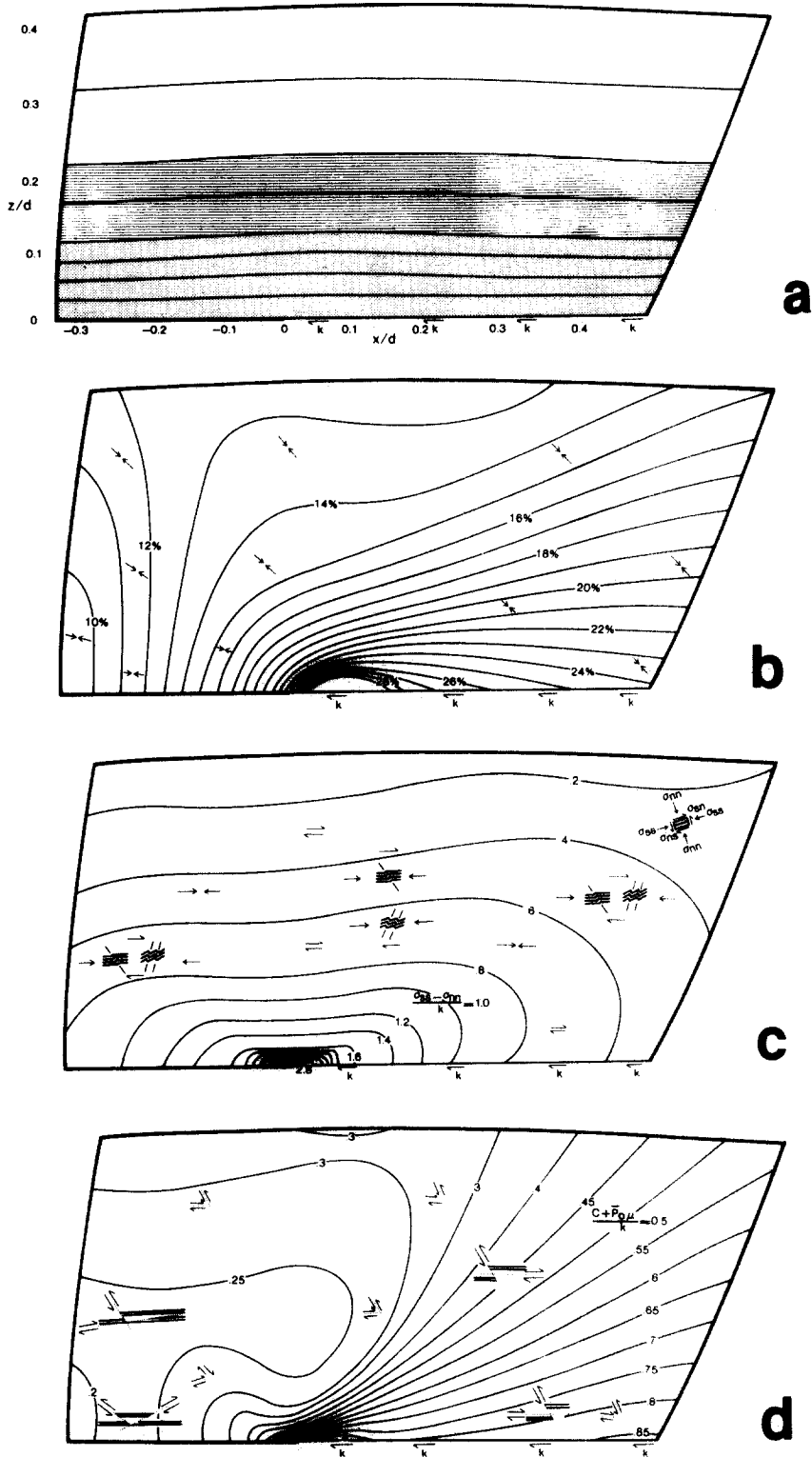


Fig. 2. Deformation in a thrust block that has been translated 0.1 unit. Markers originally horizontal and end lines originally vertical. Heavy line at base is frictionless segment. (a) Folding of passive markers. (b) Contours of finite compressive strain and magnitudes in per cent (see text for definition). Arrows show orientations of the maximum principal compressive strain. (c) Contours of normalized buckling stress  $(\sigma_{ss} - \sigma_{nn})/k$ , and senses of monoclinical kinkbands and drag folds that should form. (d) Contours of yield function  $(c + \bar{p}_0 \mu)/k$ , and senses of minor conjugate faults that should form (here  $\mu = \tan \phi$ ).

times the coefficient of friction. The friction angle is assumed to be  $30^\circ$  for all our computations. The cohesive strength and the mean pressure are normalized by  $k$ , so that the contours in Fig. 2(d) are in terms of values of the quantity, equation (4c),

$$[(C + \bar{p}_0 \tan(30^\circ))/k] = \text{yield function} \quad (5)$$

required to prevent local faulting. The larger the value, the greater the tendency for faulting according to the yield criterion, so that the contours shown in Fig. 2(d) are considered to be contours of equal tendency to fault locally. As shown in the figure, the tendency for faulting to occur is maximal near the extension of the detachment surface where adhesional drag is large. The orientations

and senses of minor faulting, according to the idealized model, are shown by inset figures in Fig. 2(d). Near the detachment surface (lower left in Fig. 2d) the ideal faults are conjugate relative to the orientation of the surface and are reverse faults. In most places one fault is a low-angle thrust fault and the other is a high-angle reverse fault dipping in the direction of transport; that is, they are 'back thrusts'.

### DEFORMATION NEAR A RAMP FAULT

Deformation patterns of viscous overburdens moving over ramp faults are shown in Figs. 3 and 4 in order to compare deformation caused solely by repetition of strata (Fig. 3a) with that caused by a combination of repetition

and of termination of slippage along the upper detachment surface (Fig. 3b). The solution for these cases is presented in the Appendix. In both cases, the inclined fault has a slope of 0.2, and the amount of horizontal translation is equal to one-fourth the height ( $S$ ) of the inclined fault segment (0.25 units). The relative amounts of layer-parallel shear are indicated by the lines bounding the figure which were vertical before deformation. Where there was zero drag, the lines are essentially vertical (Fig. 3a) and where there was drag, the line is tilted in the area of drag (right-hand side of Fig. 3b) and is nearly vertical in the area of zero drag (left-hand side).

In both examples, broad folds develop in the passive layering, but the forms of the folds are slightly different. For the ramp fault with zero drag, the fold is flat-topped with monoclinical flexures near the foot of the ramp and

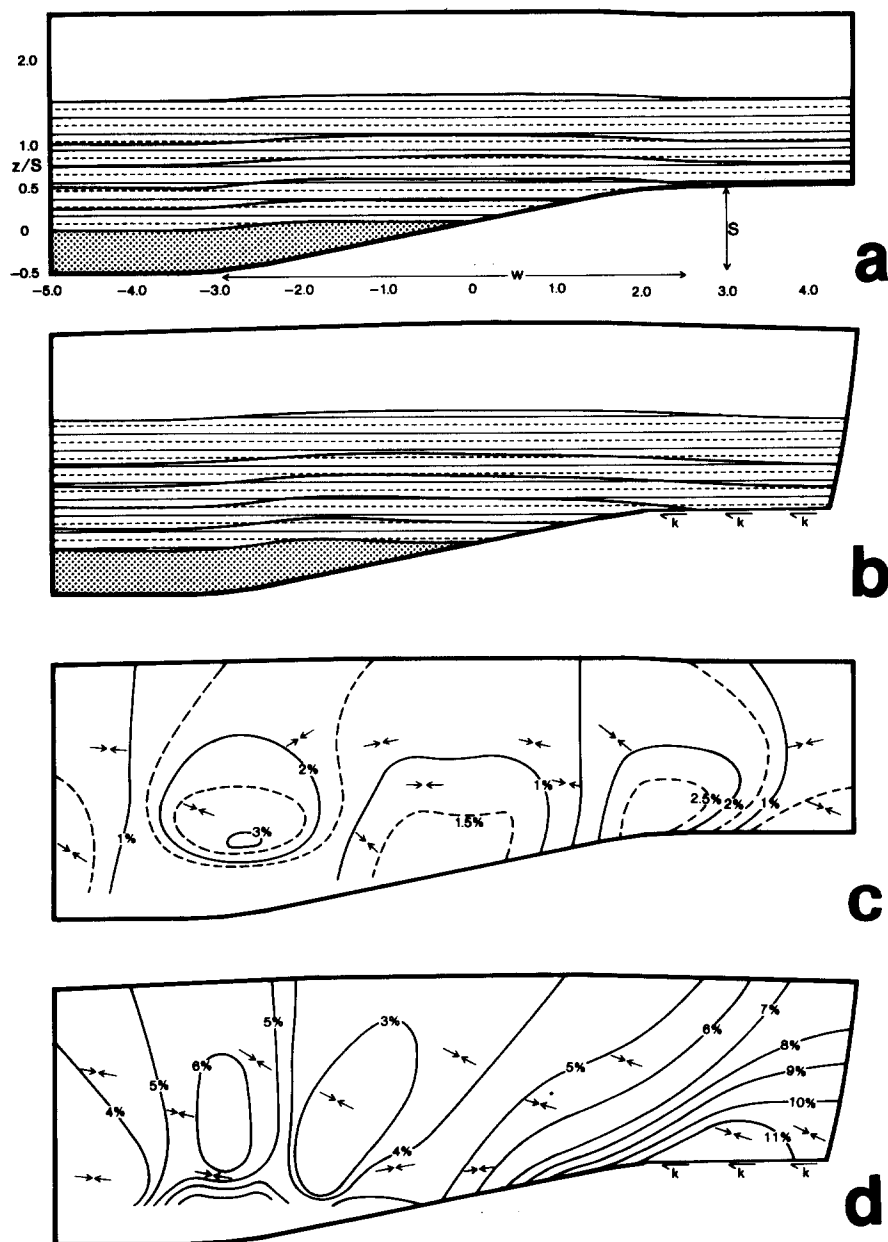


Fig. 3. Folding of passive markers and strain in the region above detachment surfaces and ramp subjected to translation of 0.5 units. (a) and (c): Frictionless detachment surfaces and ramp. (b) and (d): Terminating detachment surface and ramp. In (a) and (b), the markers were originally horizontal and the endlines vertical. (c) and (d) show the contours of finite compressive strain for (a) and (b), respectively.

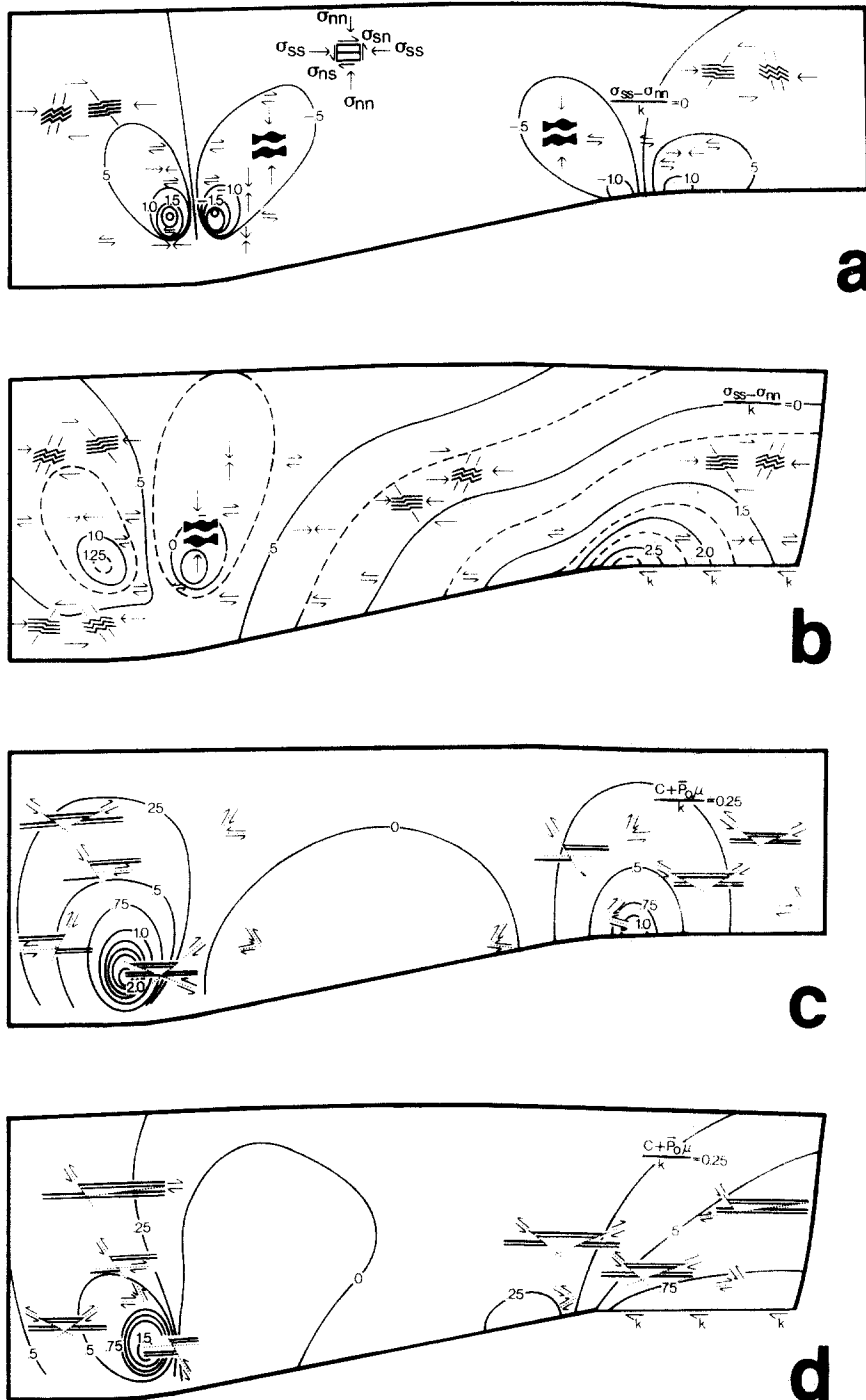


Fig. 4. Minor structures that should form in the region above detachment surfaces and ramp subjected to translation of 0.25 units. (a) and (c): Frictionless detachment surfaces and ramp. (b) and (d): Terminating detachment surface and ramp. (a) and (b) show the contours of normalized buckling stress  $(\sigma_{ss} - \sigma_{nn})/k$ . Monoclinical kink bands and drag folds may form in regions of positive stress difference, and pinch and swell structures in regions of negative stress difference. (c) and (d) show contours of yield function  $(c + \bar{p}_0 \mu)/k$  and patterns of minor conjugate faults (here  $\mu = \tan \phi$ ).

near the top of the ramp (Fig. 3a). The amplitude of folding is about the same for the passive interface at midheight on the ramp,  $z = 0$ , and for the interface at  $z = S$ . The amplitude at  $z = 2.5S$  is slightly lower, so amplitude decreases upwards at large distances from the ramp. For the terminating ramp fault (Fig. 3b), however, a small anticline develops over the foot of the ramp near midheight of the fault. At intermediate levels ( $z = 0.5S$  to  $0.75S$ ) the fold is flat-topped but slightly asymmetric. The

proximal limb is slightly steeper than the distal limb. At higher levels ( $z = 2.5S$ ), the fold is rounded and its width increases with height but the fold is still asymmetric.

The asymmetry is a result of adhesional drag along the upper detachment surface. As discussed in connection with the terminating detachment surface (Fig. 2a), the drag causes thickening in the vicinity of the termination of the ramp fault. The thickening reduces the dip of passive layering at the distal edge of the fold, whereas it has a



minor effect on passive layering at the proximal edge of the fold, so that the proximal edge is steeper than the distal.

Directions and magnitudes of maximum compressive strains are shown in Figs. 3(c) & (d). There is a zone of relatively high compressive strain in the vicinity of the foot of the ramp. For the terminating ramp fault (Fig. 3d), there is a zone of higher compression near the upper end of the ramp fault. This zone corresponds with that near the termination of a flat detachment surface (Fig. 2b).

Patterns of ideal minor folds and faults associated with a ramp fault and detachment surfaces without adhesional drag shown in Figs. 4(a) & (c), and those associated with the terminating ramp fault are shown in Figs. 4(b) & (d). The senses of asymmetry of drag folds and monoclinical kink bands are the same everywhere except near the head (Fig. 4a) or near the foot (Fig. 4b) where the sense of shear is reversed. Over the foot and head of the ramp fault connecting frictionless detachments, and over the foot of the terminating ramp fault, the layer-normal compression is larger than the layer-parallel compression, so pinch-and-swell structures or other extensional phenomena rather than folds should form there. For both ramp faults there is a strong tendency for minor faults to form near the foot of the ramp (Figs. 4c & d). For the ramp fault connecting frictionless detachments there is a secondary maximum of the yield function [equation (5)], at the top of the ramp. For the terminating ramp fault the yield function is large along the upper detachment surface, where adhesional drag is great.

The orientation and sense of displacement for the minor faults are shown in Figs. 4(c) & (d). For the case of a frictionless detachment surface and ramp, the overall pattern of minor faults is diverse, whereas for the case of a terminating ramp, back thrusts would predominate. At the foot of the ramp, for example, where the tendency for faulting is greatest, back thrusts occur in the case of a terminating detachment surface (Fig. 4d), whereas a more complex set of conjugate faults occur in the case of the frictionless detachment surface and ramp (Fig. 4c).

## DISCUSSION

We can compare the style of minor faulting derived analytically with the faulting observed in field studies of Serra (1977) and the faulting observed in experiments by Morse (1977). Serra examined beds duplicated by thrusting over ramp faults and recognized three main structural styles in the upper thrust plate (Fig. 5). Serra observed that the lower beds in the hanging wall could be subjected to thickening by imbricate faulting (Fig. 5a) or by flowage (Fig. 5b), or could be offset by backthrusts (Fig. 5c).

The thickening in the upper plate can be a result of buckle folding of a multilayer (Sherwin 1972, Wiltschko & Chapple 1977, Fletcher 1981) or of drag at the ramp (Berger & Johnson 1980). Both folding mechanisms can produce the geometry and internal deformation observed by Serra and shown in Figs. 5(a) & (b). In the case of buckling, a soft layer will thicken due to a decrease in

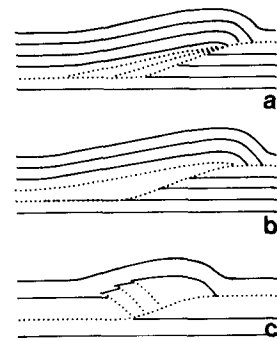


Fig. 5. Structural styles observed in ramp regions of small-scale overthrust faults (after Serra 1977). (a) Imbricate faulting in upper thrust plate. (b) Flowage in upper thrust plate. (c) Backthrusts in upper thrust plate.

vertical stress beneath anticlines, resulting in flowage (Sherwin 1972) or in faulting (Fletcher 1981). Drag at the ramp could also cause the thickening.

Serra (1977) reported that backthrusts commonly form in outcrops where the thickness of mechanical units was approximately equal to the height of the ramp. Morse (1977) experimentally produced backthrusts in folds formed by duplication of rock layers that were thrust over lubricated ramps. Morse noted that the backthrusts formed near the foot of the experimental ramp faults. In our analytical results, backthrusts also form at the foot of the ramp (Fig. 4d). However, backthrusts are possible everywhere along the terminating detachment surface (Fig. 2d). It is likely that they are a result of the frictional drag as well as the presence of a thrust ramp.

## CONCLUSION

Analysis of the deformation in the vicinity of the termination of a flat detachment surface clarifies a fundamental mechanism of folding. The mechanism is differential local thickening and layer-parallel compression produced by increased drag distally along a detachment surface. As translation continues, the fold will increase in amplitude and become asymmetric, with a steep distal limb, as a result of the mean shear caused by drag. Further, large translation can produce a nappe-shaped structure, as shown elsewhere (Berger & Johnson 1980 fig. 5d). Accordingly, mechanisms of folding include lateral termination of a detachment surface, draping over a fault (Powell 1875, Reches & Johnson 1978), by intrusion of magma (Gilbert 1877, Pollard & Johnson 1973, Koch *et al.* 1981), repetition of strata (Rich 1934, Wiltschko 1979, Berger & Johnson 1980), density instability (e.g. Ramberg 1967), and buckling (e.g. Biot 1961, Currie *et al.* 1962, Fletcher 1977). Some folds appear to have been produced predominantly by one of these mechanisms, but others apparently are caused by a combination of mechanisms.

The Burning Springs anticline is probably an example of a fold produced by several mechanisms. Rodgers (1963) presents evidence that salt in Silurian rocks pinches out in the vicinity of the anticline, so that it is possible that the

salt behaved as a viscous material with little drag and that the adjacent rock to the west behaved as a material with greater drag, much as in the idealized model of a terminating detachment surface. We would suggest that the termination of the detachment surface or a splay fault produced a low-amplitude fold which subsequently became amplified by buckling as the multilayered rocks were shortened further. Thus we suggest that the Burning Springs anticline is similar to most of the other folds on the Allegheny Plateau, west of the Valley and Ridge Province, which, according to Sherwin (1972) and to Wiltshko & Chapple (1977), are primarily a result of buckling of multilayers above zones of décollement. The most important difference would be that the Burning Springs anticline is not part of a wave train, but rather an isolated fold localized by a peculiarity of the stratigraphic section.

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## APPENDIX

Here we present the solution for velocities for the problem illustrated in Fig. 1(b), where there is a detachment surface and a ramp fault with zero drag and a higher detachment surface with adhesional drag,  $k$ . This problem is similar to those discussed elsewhere (Berger & Johnson 1980), except in the previous solutions there was zero drag on the lower and upper detachment surfaces and adhesional drag on the ramp. Further, we assumed a sinusoidal ramp fault, whereas, here we assume a straight ramp fault. For the present problem (Fig. 1b), we can identify four contributions to the velocities and stresses within the viscous fluid. One is the mean rate of translation,  $\bar{v}_x$ . Another is the velocity gradient set up by the average shear stress acting over the interface between the fluid and the rigid substratum,

$$k(z - S/2)(1 - w/d)(1/2\eta)$$

in which we have set this component equal to zero along the upper detachment surface, and  $S$  is ramp height,  $w$  is ramp width (Fig. 1b).

The third component is a perturbed velocity set up by the

interaction of the mean flow and the ramp, and the fourth component is the perturbed flow required to satisfy the stress boundary conditions along the surface. The components are arranged in the order just described in the expression for the velocity in the  $x$ -direction:

$$\begin{aligned} v_x = & \bar{v}_x + k(z - S/2)(1 - w/d)(1/2\eta) \\ & + [\bar{v}_x - kz(1 - w/d)(1/2\eta)] \sum_{m=1}^{\infty} A_m l_m (l_m z) \exp(-l_m z) \cos(l_m x) \\ & + (1/2\eta) \sum_{n=1}^{\infty} (R_n/l_n) [(l_n z - 1) \exp(-l_n z) \cos(l_n x) \\ & - \sum_{m=1}^{\infty} (A_m/2) \{(l_q + l_r(l_q z - 1)) \exp(-l_q z) \cos(l_q x) \\ & \pm (l_r \pm l_q(l_r z \mp 1)) \exp(\mp l_r z) \cos(l_r x)\}] \end{aligned} \quad (A1)$$

$$\begin{aligned} v_z = & - [\bar{v}_x - kz(1 - w/d)(1/2\eta)] \sum_{m=1}^{\infty} A_m l_m (1 + l_m z) \exp(-l_m z) \\ & \sin(l_m x) - (1/2\eta) \sum_{n=1}^{\infty} (R_n/l_n) [l_n z \exp(-l_n z) \sin(l_n x) \\ & - \sum_{m=1}^{\infty} (A_m/2) \{l_q(1 + l_r z) \exp(-l_q z) \sin(l_q x) \\ & + l_r(1 \pm l_q z) \exp(\mp l_r z) \sin(l_r x)\}]. \end{aligned} \quad (A2)$$

Here

$$A_m = (4S/w) d (1/m\pi)^2 \cos[m\pi(1 - w/d)/2]; \text{ for all } m \quad (A3)$$

$$R_n = (2k/n\pi) \sin[n\pi(1 - w/d)/2]; \text{ odd } n \quad (A4)$$

$$q = [(m/n) + 1]; r = [(m/n) - 1]; l_q = ql_n; l_r = rl_n. \quad (A5)$$

The upper signs in eqs. (A1) and (A2) are selected for  $m > n$ , and the lower signs are selected for  $m < n$ .

The derivation of expressions for stresses and the drag parameter,  $K$ , is similar to that discussed in connection with equations (2)–(4) in the main text, but the expressions are much longer and so will not be presented. The expressions for stresses and velocities converge for large  $m$  and  $n$  if  $z$  is positive; that is, for points in the fluid that lie above midheight on the ramp. The expressions for stresses converge more slowly than those for velocities, because the Fourier components of the velocities converge according to  $1/m$  and  $1/n^2$  whereas those for the stresses converge according to  $1$  and  $1/n$ . Equations (A1) and (A2) diverge for  $z$  negative, that is, for points in the fluid that lie below midheight on the ramp. We have elected not to compute velocities and stresses for points below midheight on the ramp, but we could have performed the computations by using plus and minus options as we did for parameter  $r$  in equations (A1) and (A2).